Dimension of Product Spaces (Shape Theory and Topological Spaces)

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Dimension of product spaces

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For a topological space X, we denote by dim X the covering dimension of X in the sense of Katetov. Consider the following relations.

(I) $\dim (X \times Y) \leqq \dim X + \dim Y$

(II) $\dim (X \times Y) > \dim X$

(III) $\dim (X \times Y) = \dim X + \dim Y$

For the relation (I), B.A.Pasynkov proved the following beautiful theorem.

Theorem 1. [4] *If the product space* $X \times Y$ *is rectangular, then the relation* (I) *holds.*

This is a very strong theorem. In almost all cases where it was proved that the relation (I) held, the product $X \times Y$ was rectangular. Hence the following Yajima's theorem is interesting.

Theorem 2. [5] *Let* X *be a collectionwise normal space which has a* σ*-closure-preserving closed cover by* m*-compact sets and* Y *be a subparacompact space with* $\chi(Y) \leqq m$. *If* $X \times Y$ *is normal, then* (I) *holds.*

Note that $X \times Y$ is not generally rectangular in Theorem 2.

Anderson and Keisler [1] constructed a separable metric space X_n, $n = 1,2,\ldots$, with the following property: $\dim X_n^k = n$ for $k = 1,2,\ldots,\infty$, where X_n^k is the k-fold product of X_n. Thus the following problem is interesting.

Problem 1. For a paracompact space X and a compact space Y with $\dim Y > 0$, does the relation (II) hold ?

A partial answer to Problem 1 holds.

Theorem 3 [3] If X is paracompact and Y is compact such that $\dim Y > 0$ and the 1-dimensional Čech cohomology group $\check{H}^1(Y) = 0$, then (II) holds.

Corollary. If Y is a non-degenerate ASR, that is, a compact space with trivial shape, then (II) holds for every paracompact space X.

Problem 2. (K.Borsuk) If Y is a compact ANR, then does (III) hold for every paracompact space X ?

It is known [2] that if Y is a 2-dimensional compact ANR, then (III) holds for every paracompact space X.

Problem 3. If Y is a continuous curve, then does (III) hold for every paracompact space X.

The following partial answer to Problem 3 is known.

Theorem 4 [3] If Y is a movable curve, then (III) holds for every paracompact space X.

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